Time Varying Regression and Changing Correlation

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Preview

The aim is to examine **time varying regression models** from the point of view of a **time varying joint distribution** (*ie* - time varying correlations and variances)

- We would argue it is a more logical starting point
- More satisfactory properties
- Can be easily implemented using the DCS/GAS methodology
- Other issues such as testing for parameter constancy follow naturally

Talk Outline

1. Standard TVP Regression
2. Using the Joint Distribution
3. Testing
4. Simulation Study - Testing and Estimation
5. Application - Shanghai/NYSE Convergence

Motivation for TVP Regression

- DCS implies we are all on board with TVP to some extent
- Evidence of wide spread parameter instability from Stock and Watson (1996, JBES) using 76 macroeconomic and financial time series.
- Lots of examples appearing in the literature - Cogley and Sargent (2002, 2005, 2010), Ang and Chen (2007), Dangl and Halling (2012), etc....

Standard Methods

Time-varying parameters are usually modeled in a regression framework by letting the coefficient of an explanatory variable change over time. Thus

$$y_t = \beta_t x_t + \varepsilon_t, \quad t = 1, ..., T,$$

where β_t follows a stochastic process, such as an AR(1) or random walk; see Harvey (1989, p. 408-11) and the test procedure proposed by Nebeya and Tanaka (1988).

- This model is valid if the explanatory variable is non-stochastic, for example, if it is a function of time. If the explanatory variable is stochastic then it needs to be independent of β_t and ε_t in all time periods.
- If x_t is a linear process, then y_t will, in general, be nonlinear.
- Concerns about the path followed by the dependent variable when the explanatory variable is integrated; see the discussion in Harvey (1989, p. 409-10).

Regression from the Joint Distribution

Suppose that y_{1t} and y_{2t} are jointly normal with zero means and covariance matrix

$$\Sigma = \begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix}$$

This implies

$$y_{1t} \mid y_{2t} = \beta y_{2t} + \varepsilon_t, \quad \varepsilon_t \sim NID\left(0, \sigma_\varepsilon^2\right), \qquad t = 1, ..., T,$$

where

$$\beta = \sigma_{12}/\sigma_2^2 = \rho\sigma_1/\sigma_2$$

and ε_t is distributed independently of y_{2t} with variance $\sigma_\varepsilon^2 = \sigma_1^2 - \rho^2\sigma_2^2$.
NOTE: If Σ is time dependent then β is no longer **weakly exogenous** as in the static case studied by Engle, Hendry and Richard (1983).

What to do?

We propose to model the time varying joint distribution with the DCS/GAS approach and then derive the implied time varying regression model.

- The Simple Bivariate Case

$$\begin{bmatrix} y_t \\ x_t \end{bmatrix} \sim N\left(\mu = \begin{bmatrix} \mu_y \\ \mu_x \end{bmatrix}, \Sigma_t = \begin{bmatrix} \sigma_{yt}^2 & \rho_t \sigma_{yt} \sigma_{xt} \\ \rho_t \sigma_{yt} \sigma_{xt} & \sigma_{xt}^2 \end{bmatrix}\right)$$

with link functions

$$\begin{aligned} \sigma_{it}^2 &= exp(2\lambda_{it}) \\ \rho_t &= \frac{\bar{g}}{g} = \frac{\frac{e^{\gamma_t} - e^{-\gamma_t}}{2}}{\frac{e^{\gamma_t} + e^{-\gamma_t}}{2}} \end{aligned}$$

The dynamics for the time varying parameters in the joint distribution are given by

$$\begin{aligned}
\gamma_{t+1|t} &= \omega_1(1-\phi_1) + \phi_1\gamma_{t|t-1} + \kappa_1 v_{1t} \\
\lambda_{yt+1|t} &= \omega_2(1-\phi_2) + \phi_2\lambda_{yt|t-1} + \kappa_2 v_{2t} \\
\lambda_{xt+1|t} &= \omega_3(1-\phi_3) + \phi_3\lambda_{xt|t-1} + \kappa_3 v_{3t}
\end{aligned}$$

The scaled score is given by

$$\begin{aligned}
v_{1t} &= \frac{2(y_t-\mu_y)(x_t-\mu_x)}{\sigma_{yt}\sigma_{xt}} g_t^2 - \left[\frac{(y_t-\mu_y)^2}{\sigma_{yt}^2} + \frac{(x_t-\mu_x)^2}{\sigma_{xt}^2}\right] g_t\bar{g}_t \\
v_{2t} &= 0.5\left(\frac{(y_t-\mu_y)^2}{\sigma_{yt}^2} - 1\right) \\
v_{3t} &= 0.5\left(\frac{(x_t-\mu_x)^2}{\sigma_{xt}^2} - 1\right)
\end{aligned}$$

- The method extends beyond the bivariate case - CKL (2011, JBES)
- Can explicitly account for heavy tails
- Beta-t-EGARCH enriches dynamics
- Issues such as model selection and diagnostics dealt with from implied conditional distribution and errors.

Testing Approach - Time Varying Correlation

We suggest an alternative to the LM test of Nyblom (1989) based on the Ljung-Box test.

$$Q(\hat{\gamma}) \quad = \quad T(T+2)\sum_{\tau=1}^{P} r_d^2(\tau)/(T-\tau)$$

where $r_d(\tau)$ is the τ^{th} autocorrelation of the scores evaluated at the MLE of the static parameter.

- Test stat is Chi-Squared with df $= m$, the No. of fixed parameters in the dynamic equations.
- Test should have greater power when the parameters aren't random walks.
- The test can be extended to the multivariate setting along the lines of Hotelling (1980) using results from CKL (2011).

Simulation Study

Simple bivariate model with changing correlation driven by DCS functional form.

- Sample sizes of $T = 200, 500, 1000$,
- $\omega = \{0.5, -0.2\}$, $\phi = \{0.1, 0.5, 0.8, 0.95, 0.98, 1\}$ and $\kappa = \{0, 0.1, 0.02, 0.3, 0.04\}$
- Estimates evaluated with the Bias, RMSE and interquartile range (IQR)
- 1000 draws for estimation and 5000 for testing
- Number of lags for the Ljung-Box test was $max(20, \sqrt{T})$
- Significance level was set to 0.05% for all tests.

Estimation Results

Table : **Simulation Results for ω**

κ		0.02				0.04			
ϕ		0.8		0.95		0.8		0.95	
ω	T	0.5	-0.2	0.5	-0.2	0.5	-0.2	0.5	-0.2
BIAS	200	0.16	-0.12	0.13	-0.02	0.15	-0.09	0.03	0.02
x10	500	0.02	-0.04	0.02	-0.01	0.03	-0.02	0.02	-0.03
	1000	0.01	-0.04	0.02	-0.03	0.01	0.00	0.01	0.01
RMSE	200	1.11	1.34	1.28	1.37	1.10	1.24	1.46	1.47
x10	500	0.56	0.70	0.71	0.74	0.57	0.64	0.90	0.90
	1000	0.35	0.44	0.48	0.52	0.38	0.43	0.58	0.56

Estimation Results

Table : **Simulation Results for ϕ**

κ		0.02				0.04			
ϕ		0.8		0.95		0.8		0.95	
ω	T	0.5	-0.2	0.5	-0.2	0.5	-0.2	0.5	-0.2
BIAS	200	-1.38	-1.52	-2.41	-2.81	-1.62	-1.60	-1.85	-2.23
x10	500	-1.55	-1.41	-1.92	-2.30	-1.39	-1.41	-0.93	-1.05
	1000	-1.38	-1.44	-1.16	-1.42	-0.90	-1.24	-0.27	-0.38
RMSE	200	3.76	4.18	4.13	4.75	3.75	3.96	3.46	4.11
x10	500	3.64	3.88	3.57	4.26	3.24	3.55	2.24	2.55
	1000	3.42	3.82	2.67	3.15	2.51	3.15	0.22	0.130

Estimation Results

Table : **Simulation Results for κ**

κ		0.02				0.04			
ϕ		0.8		0.95		0.8		0.95	
ω	T	0.5	-0.2	0.5	-0.2	0.5	-0.2	0.5	-0.2
BIAS	200	-0.15	-0.29	-0.07	-0.17	0.00	-0.14	0.09	-0.06
x10	500	0.06	-0.03	0.07	0.00	0.13	0.03	0.13	0.06
	1000	0.09	0.01	0.09	0.05	0.12	0.04	0.11	0.05
RMSE	200	1.01	1.06	0.90	0.97	0.95	1.02	0.79	0.87
x10	500	0.49	0.50	0.41	0.47	0.45	0.46	0.37	0.37
	1000	0.31	0.33	0.24	0.28	0.31	0.31	0.22	0.20

Empirical Size of Parameter Constancy Tests

Table : **Size Comparison for Parameter Constancy Tests**

	ϕ					
	0.1	0.5	0.8	0.95	0.98	1
T=200						
N	0.05	0.05	0.05	0.05	0.05	0.04
Q	0.05	0.05	0.05	0.06	0.05	0.05
T=500						
N	0.05	0.04	0.04	0.05	0.06	0.06
Q	0.05	0.05	0.06	0.05	0.06	0.05
T=1000						
N	0.04	0.05	0.05	0.05	0.05	0.05
Q	0.06	0.05	0.06	0.05	0.05	0.06

Note: Nominal size is 5%. N and Q are the Nyblom and Ljung-Box tests respectively. $\omega = 0.5$ and $\kappa = 0$.

Power Comparison for Parameter Constancy Tests

	ϕ					
	0.1	0.5	0.8	0.95	0.98	1
T=200, $\kappa = 0.02$						
N	0.05	0.06	0.08	0.16	0.24	0.30
Q	0.06	0.06	0.07	0.09	0.11	0.11
T=1000, $\kappa = 0.02$						
N	0.06	0.07	0.09	0.24	0.49	0.91
Q	0.06	0.07	0.11	0.30	0.53	0.86
T=200, $\kappa = 0.04$						
N	0.07	0.08	0.12	0.32	0.45	0.60
Q	0.07	0.07	0.10	0.21	0.29	0.38
T=1000, $\kappa = 0.04$						
N	0.07	0.08	0.15	0.46	0.80	0.99
Q	0.11	0.14	0.29	0.79	0.96	1

Note: Significance level is 5%. N and Q are the Nyblom and Ljung-Box tests respectively. $\omega = 0.5$. 5000 replications

Application - Shanghai/NYSE convergence

Revisiting Chow, Liu and Niu (2011) "Co-movement of Shanghai and New York stock prices by time-varying regressions", *Journal of Comparative Economics*.

Aim: To examine the integration of these two markets over time.

- Extend Chow *et al* (2011)

$$\begin{aligned} r_t^n &= \alpha + \beta_t Z_t + \sigma e_t \\ \beta_t &= \beta_{t-1} + u_t \end{aligned}$$

where $Z_t = [r_t^s \quad r_{t-1}^n \quad r_{t-1}^s]'$

$$\begin{aligned} Z_t &= [r_t^s]' \\ Z_t &= [r_t^s \quad r_{t-1}^n]' \\ Z_t &= [r_t^s \quad r_{t-1}^n \quad r_{t-1}^s]' \end{aligned}$$

Score Approach

Based on TV multivariate t-distribution.

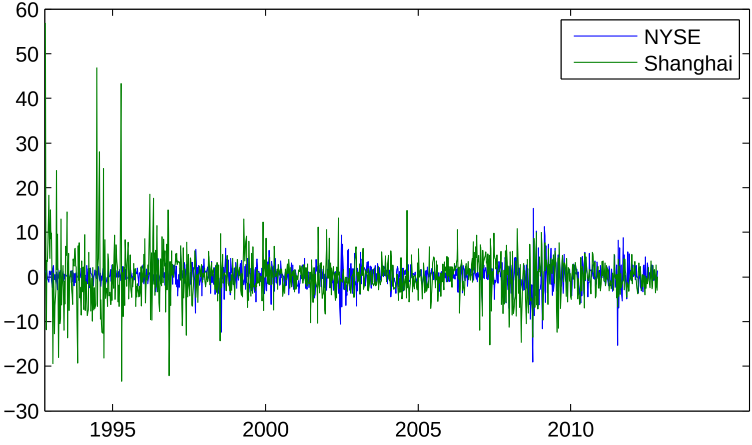
$$\begin{bmatrix} r_t^n \\ r_t^s \end{bmatrix} \sim t\left(\nu, \mu = \begin{bmatrix} \mu_n \\ \mu_s \end{bmatrix}, \Sigma_t = \begin{bmatrix} \sigma_{it}^2 & \rho_1 t\sigma_{yt}\sigma_{xt} \\ \rho_t\sigma_{yt}\sigma_{xt} & \sigma_{xt}^2 \end{bmatrix}\right)$$

Captures heavy tails and GARCH effects found in stock returns.

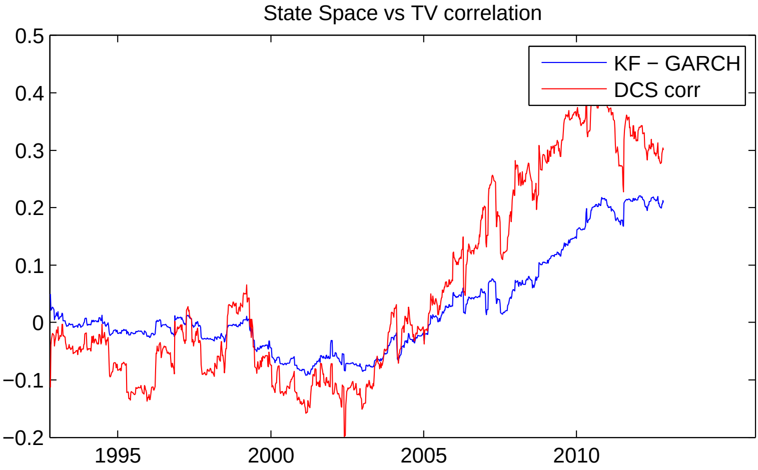
$$\begin{aligned} \gamma_{t+1} &= \gamma_t + \kappa_1 v_{1t} && (1) \\ \lambda_{nt+1} &= \omega_2(1-\phi_2) + \phi_2\lambda_{nt} + \kappa_2 v_{2t} && (2) \\ \lambda_{st+1} &= \omega_3(1-\phi_3) + \phi_3\lambda_{st} + \kappa_3 v_{3t} && (3) \end{aligned}$$

Shanghai/NYSE Data

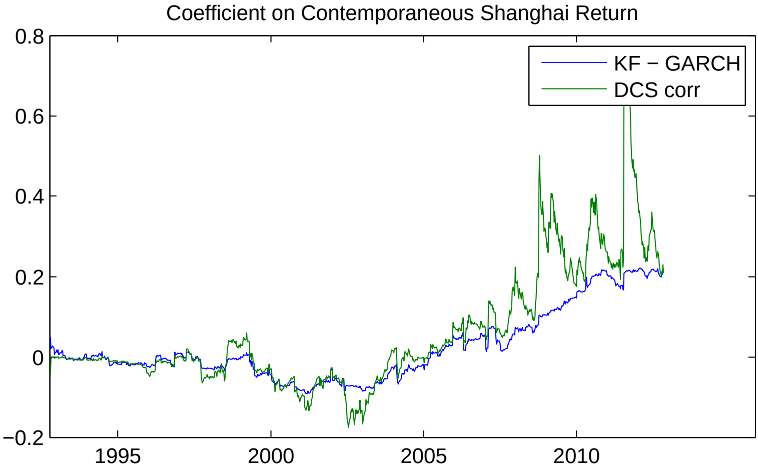
Weekly returns on Shanghai & NYSE indices from 11/1992 - 11/2012.



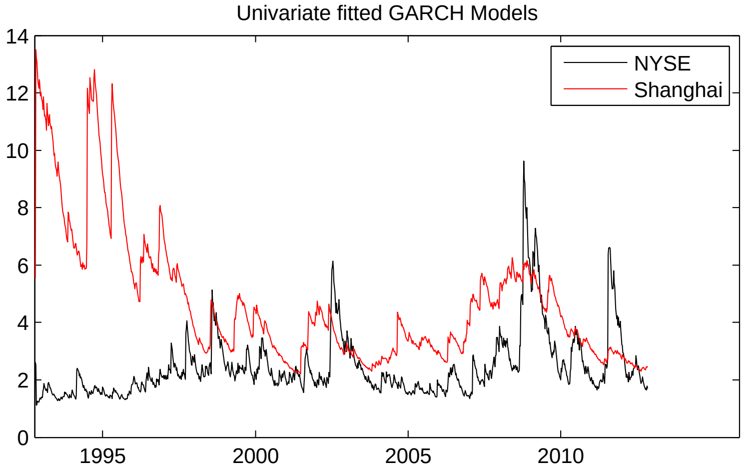
NYSE = f(Shanghai)

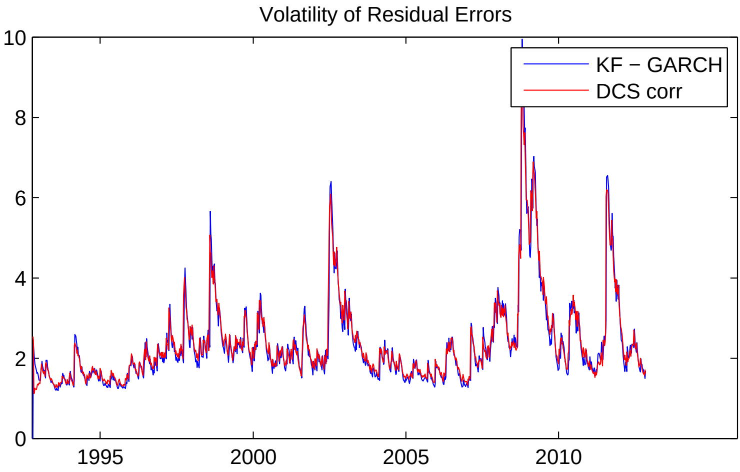


NYSE = f(Shanghai)



GARCH Effect Driving TV Coefficient




Volatility of Residual Errors
KF − GARCH
DCS corr
10
8
6
4
2
0
1995
2000
2005
2010

Conclusion

- Examined TVP regressions based on a TV joint distribution.
- More satisfactory properties.
- Can be easily implemented using the DCS/GAS methodology.
- Demonstrated testing approach to TVP which can be further developed.